LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – NOVEMBER 2015

MT 3810 - TOPOLOGY

Date : 03/11/2015 Dept. No. Max. : 100 Marks
Time : 09:00-12:00

Answer All questions. Each question carries 20 marks.

I a) i) Define and give examples for metric and pseudo metric spaces.

OR

a) ii) Using an example, show that continuity does not imply uniform continuity.

b) Let (X,d) be a metric space. Define $d_1(x,y) = d(x,y)/ [1+ d(x,y)]$ Prove that d_1 is a metric on X.

c) State and prove Baire's theorem.

OR

d) Let X be a complete metric space and Y be a subspace of X. Then prove that Y is complete iff Y is closed.

e) State and prove Cantor's intersection theorem.

II a) i) Prove that any closed subspace of a compact space is compact.

OR

a) ii) Prove that any continuous image of a compact space is compact.

b) State and prove Tychonoff's theorem.

c) State and prove Lebesque covering lemma.

OR

d) State and prove Ascoli's theorem.

III a) i) Prove that every compact subspace of a Hausdorff space is closed.

OR

a) ii) Prove that the product of any non-empty class of Hausdorff spaces is a Hausdorff space.

b) State and prove Urysohn's embedding theorem.

OR

c) State and prove Tietze extension theorem.

IV a) i) Prove that the product of any non-empty class of connected spaces is connected.

OR

a) ii) Let X be a Hausdorff space. If X has an open base whose sets are also closed then prove that X is totally disconnected.

b) Proving the necessary results, show that the range of a continuous real function defined on a connected space is an interval.

c) Prove that a topological space X is disconnected iff there exists a continuous mapping of X onto the discrete two point space {0,1}.

OR

d) If X is an arbitrary topological space, then prove the following:

(i) each point in X is contained in exactly one component of X

(ii) each connected subspace of X is contained in a component of X

(iii) a connected subspace of X which is both open and closed is a component of X and

(iv) each component of X is closed.

V. a) i) Prove that X_∞ is compact

OR

a) ii) Prove that X_∞ is Hausdorff.

b) State and prove the real Stone-Weierstrass theorems.

OR

c) State and prove Weierstrass Approximation theorem.

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